A mathematical framework for neural dynamics

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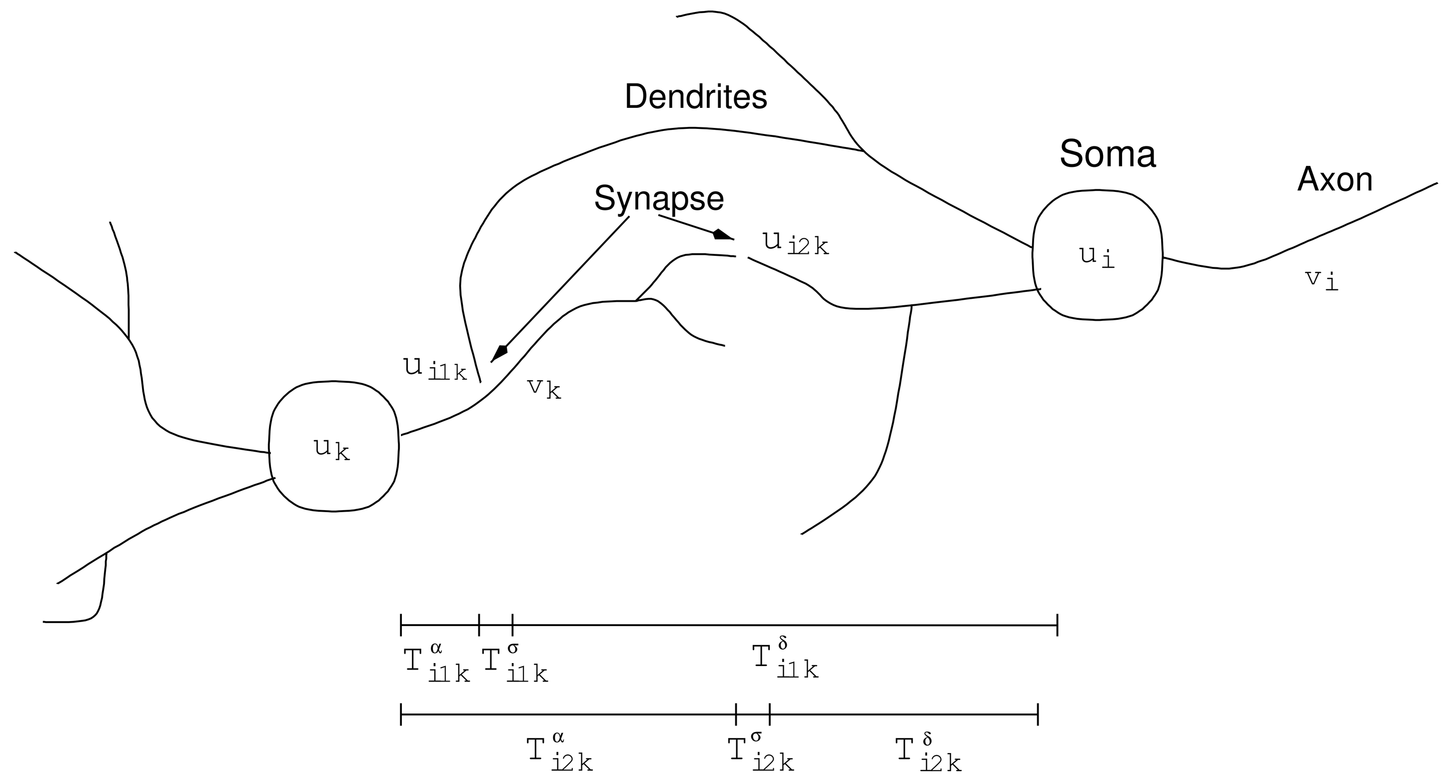
Introduction

The central nervous system of animals and human beings is without doubt one of the most fascinating organism. Although many highly specialized types of neurons exist the layout is always the same: soma, axon, and dendrites with synaptic connections, see figure below.

This work presents a flexible and efficient modelling framework for:

- large populations with arbitrary geometry
- different synaptic connections with individual dynamic characteristics
- cell specific axonal dynamics



Axonal Dynamics

Say we have $i = 1, \dots, m$ neurons. The action potenial of neuron i can be described in the following form (e.g. Hodgkin-Huxley):

$$\begin{aligned} \dot{\mathsf{v}}_i(t) &= \Phi_i(\mathsf{v}_i(t), \mathsf{w}_i(t)) + \mathsf{u}_i(t) \\ \dot{\mathsf{w}}_i(t) &= \Psi_i(\mathsf{v}_i(t), \mathsf{w}_i(t)) \end{aligned} \tag{1}$$

$\mathsf{v}_i(t) \in \mathbb{R}$ is the membrane potential at the axon initial segment, while $\mathsf{w}_i(t) \in \mathbb{R}^d$ $(d \in \mathbb{N})$ describes auxiliary variables and $\mathsf{u}_i(t)$ is the total post-synaptic potential.

Net Dynamics

The total post-synaptic potential u_i is the sum of the the incoming post-synaptic signals u_{ijk}

$$\mathsf{u}_i(t) = \sum_{k=1}^{m} \sum_{j=1}^{n_{ik}} \delta_{ijk}\, u_{ijk}(t - T^{\delta}_{ijk}) \quad i = 1, \dots, m \tag{2}$$

n_{ik} is the number of synapses between neuron i and neuron k.

δ_{ijk} is a dendritic signal easing factor.

T^{δ}_{ijk} describes the time delay for the signal propogation along the dendrite.

Synaptic Dynamics

The post-synaptic potential u_{ijk} of neuron i generated by the pre-synaptic neuron k at the synapse j is modeled as:

$$u_{ijk}(t) = \int_{-\infty}^{t} q_{ijk}\, h_{ijk}(t - t')\, g_{ijk}(\mathsf{v}_k(t' - (T^{\alpha}_{ijk} + T^{\sigma}_{ijk})))\, dt' \tag{3}$$

$u_{ijk}(t)$ is the post-synaptic potential.

q_{ijk} represents the strength of the synaptic connection, the sign of q_{ijk} decides if the synapse is excitatory $(+)$ or inhibitory $(-)$.

$h_{ijk}(t)$ is an appropriate temporal weighting function modelling the dynamic membran properties.

g_{ijk} is a monotonically increasing, nonnegative, and bounded function which describes the transduction between the pre- and postsynaptic potential.

$T^{\alpha}_{ijk}, T^{\sigma}_{ijk}$ are time delays modelling the signal propagation along the axon and synapse respectively.

Example

The model presented here has been used successfully by several authors for neural nets of small and large population, see [2, 3, 5, 1].

Here we present the case of a synchronized population of inhibitory cells[1]:

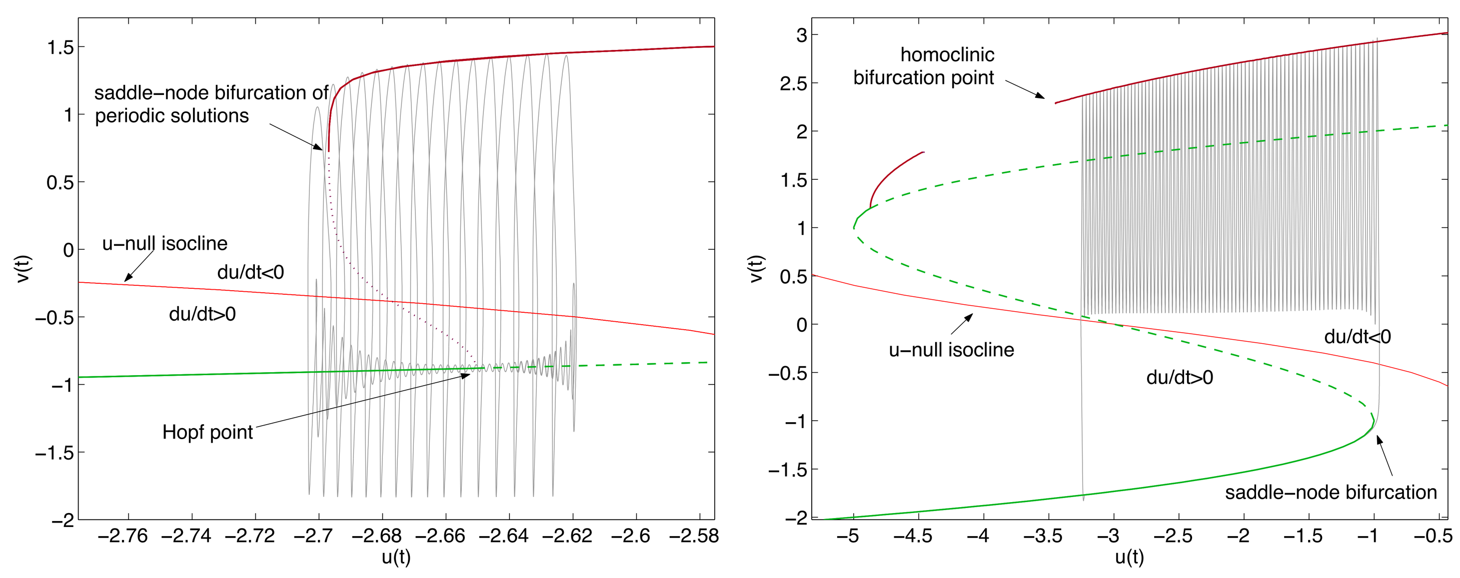
$$\begin{aligned} \dot{\mathsf{u}}(t) &= \alpha(-\mathsf{u}(t) + qg(\mathsf{v}(t - T)) + u_0) \\ \dot{\mathsf{v}}(t) &= \mathsf{w}(t) - \varphi(\mathsf{v}) + \mathsf{u}(t) \\ \dot{\mathsf{w}}(t) &= \psi(\mathsf{v}(t)) - \mathsf{w}(t) \end{aligned} \tag{4}$$

We study three different aspects:

- the influence of the time constant α
- the time delay T
- the choice of the oscillator for the axonal dynamics

Oscillator

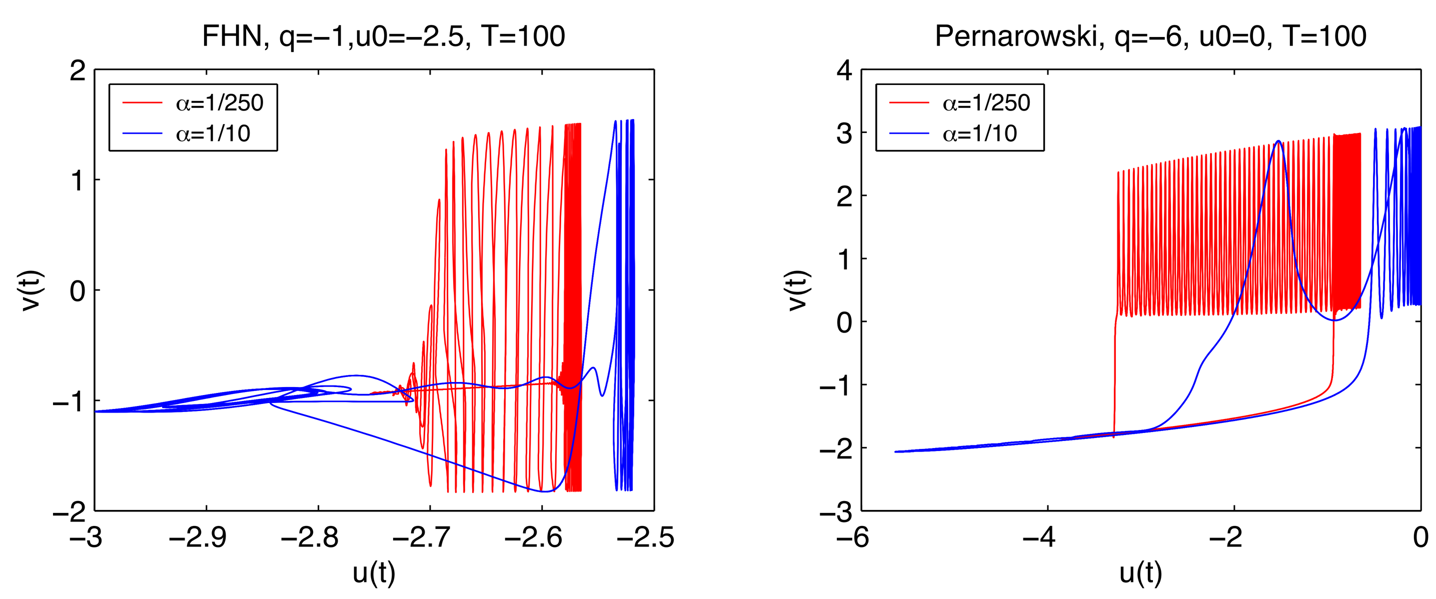
The model (1-3) and (4) allows to study different oscillators, e.g. the ones from FitzHugh-Nagumo[2] (left below) and Pernarowski[3] [4] (right below) with different dynamic characteristics.



Influence of the time delay

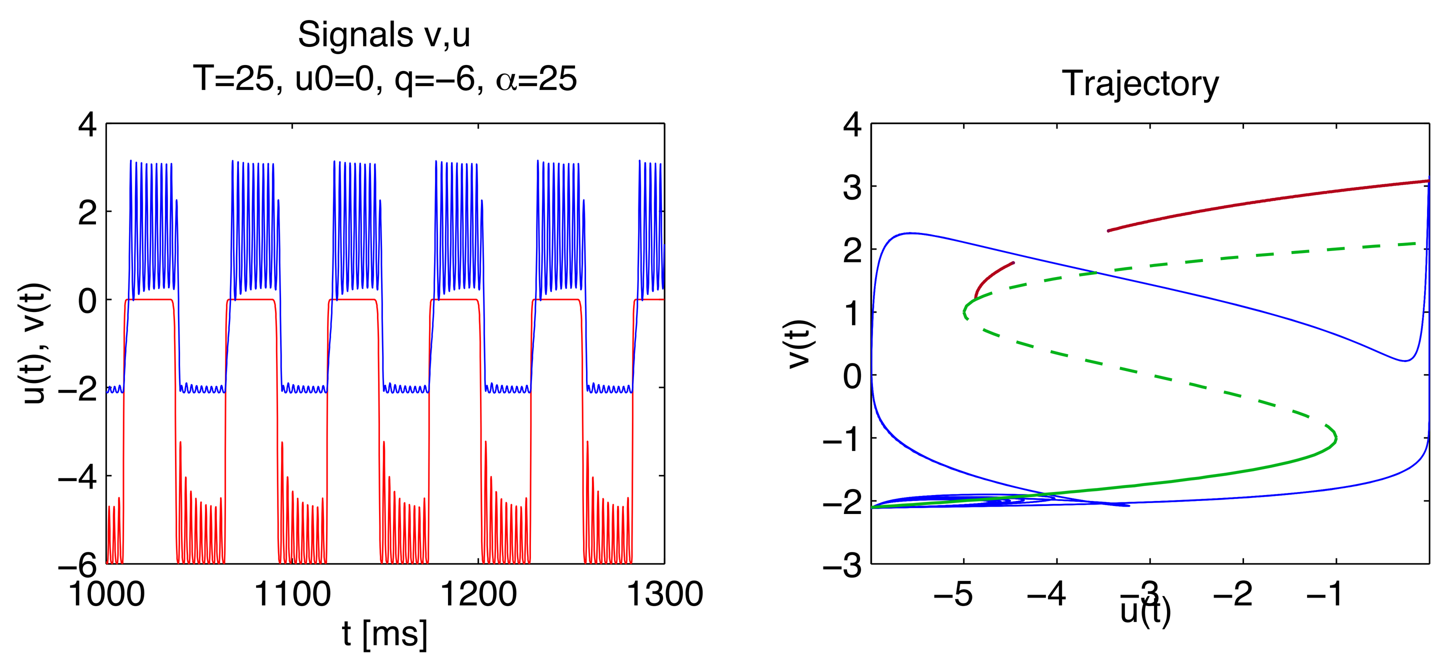
Time delay changes the frequency

The parameter of the time constant α and the time delay T have an influence on bursting frequency and the action potential threshold.



Time delay enables bursting

The following artificial example with the Pernarowski oscillator shows that burst can be generate even if $\alpha > 1$, if a time delay $T > 0$ is introduced.



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References

[1] F. Giannakopoulos A. Gail. Bursting in a model with delay for synchronised neurons. contributed poster,Section: Neural systems and the brain, this conference. Date/Time: Tue, July 19 AND Thu, July 21 / 19:00 - 22:00 Room: Poster Gallery, No. 6-8.

[2] Markus Gesmann. Modeling and analysing neuronal dynamics. Master's thesis, University of Cologne, 2002. supervised by Fotios Giannakopoulos and Tassilo Küpper.

[3] F. Giannakopoulos, C. Hauptmann, U. Bihler, and H.J. Luhmann. Epiletiform activity in a neocortical network: a mathematical model. *Biological Cybernetics*, 85:257–268, 2001.

[4] M. Pernarowski. Fast subsystem bifurcations in slowly varying lienard system exhibiting bursting. *SIAM J. App. Math.*, 54:841–832, 1994.

[5] H. Liljenstrøm Y. Gu, G. Halmes and B. Wahlund. A cortical network model for clinical eeg data analysis. *Neurocomputing*, 58-60:1197–1196, 2004.

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[1] $n = m = 1$, $\delta = 1$, $h(t) = \alpha e^{-\alpha t}$ for $t \geq 0$, 0 otherwise, $g(v) = \frac{1}{1+\exp(-4v)}$, u_0 is the resting cell potential

[2] φ cubic, ψ linear

[3] φ and ψ cubic